Reconstructing economics: Agent based models and complexity

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In this paper we argue that a view of the economy as a complex interactive and adaptive system allows one to explain phenomena such as “phase transitions” which are not consistent with the equilibrium models of standard models. A growing strand of economists is now following a different methodology based upon the analysis of systems with many heterogeneous interacting agents. We explain how Agent Based Models (ABM), and, in particular Agent based Computational Economics (ACE), and Analytically Solvable Heterogeneous Interacting Agent (ASHIA) models based on statistical physics or Markov chains can be used to deal with economies in which direct interaction between agents is important. This interaction leads to empirical regularities, which emerge from the system as a whole and cannot be identified by looking at any single agent in isolation: these emerging properties are, according to us, the main distinguishing feature of a complex system. In this way economics can free itself from the limitations of the static equilibrium approach, the use of the implausible Representative Agent approach and analyze the, possibly out of equilibrium, dynamics which seem more consistent with observed empirical data. The complexity approach is a very challenging line of research whose empirical results are very promising. Modeling an agent-based economy, however, remains a complex and complicated adventure.

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1. Introductory remarks

In 1803 Louis Poinsot, a French physicist, wrote a highly successful book, Elements de Statique, which was destined to have practical and social influences far beyond those he had ever imagined. This influence is essentially due to the work of Leon Walras who took as the basis for his theory, Poinsot’s system of simultaneous and interdependent equations. However Walras assumed that the necessary adjustment of prices was done by some external agent, and although not explicitly referred to by Walras himself\footnote{Corresponding author.} this has come to be thought of as an auctioneer, a device that allows one to reduce the economic agents to atoms, devoid of any phenomenon of learning or strategic behav-

\footnote{Leonardo Bargigli and Gabriele Tedeschi are partly responsible for what you read.}

\footnote{See Walker (1996).}
ior. We were, around 1870, in the midst of classical mechanics and reductionism. The physics of interacting microelements was still to come, and economics found it trapped in the “equilibrated” view of Walras. From there on, the union of axiomatization and non-falsifiability led to the Lakatosian degeneration of the paradigm of standard economic theory to the state that we find it in today.

The increasingly evident crisis of the dominant paradigm is manifested also through the “support” for alternative approaches offered by other disciplines, from biology to chemistry, and from neurology to physics. The contribution made to economic research by econophysics consists essentially in an approach that makes extensive use of experimental methodology and that operates on the data, often of very high frequency, related to the real markets, deriving empirical regularities (not laws) and phenomenological models. Statistical physics provides useful tools to analyze systems composed of many heterogeneous agents (atoms) whose interactions are governed by microscopic laws. Unlike economic agents, however, atoms are not able to adopt either learning or strategic behavior, both of which are derived from interaction. However, once the rules which govern the learning process or the choice of strategies are specified the tools can be applied.

Modeling any system, as soon as the logical structure of the author of the model is specified obviously involves a loss of information. As has been famously observed, “the map is not the territory”. What we propose below is no exception, although it has, in our opinion, the great and significant advantage of having an immediate and intelligible interpretation. Macroeconomic theory can be divided, for our purposes, into:

- A branch which does not deal with direct interactions between agents, and which bypasses the problem by making the assumption of
  - A representative agent (RA), or of
  - Heterogeneous agents with a fixed distribution of characteristics who make decisions independently of each other.
- A branch analyzing the issue of heterogeneous interacting agents (HIA) using Agent Based Models (ABM), which can be separated into two approaches:
  - Agent based Computational Economics (ACE),
  - Analytically Solvable HIA (ASHIA) models based on statistical physics or Markov chains.4

3 We would argue that the epistemological status of a hard sciences differs radically from that of a soft sciences. Consider, for example, a round of betting on “how long does it take for a certain object pushed out of a plane to fall on the floor”. Suppose now that the law of falling bodies is revealed to all punters. There are no more bets, but the physical law remains valid: in the soft sciences, the discovery of a “law” means that the regularity disappears through learning. Thus the very notion of the term “law” is quite different in the two cases.

4 With the introduction of the learning mechanism, Landini et al. (2012b) present a model that goes beyond the tools of statistical mechanics, and open the way for its application to social science, and to
The difference between the two branches is rooted in the assumptions about information. If information is complete and agents are small and non-strategic, there is no room for direct non-market interaction because agents have no incentive to increase their information through interaction. Even in this context a number of consequences follow: there is no coordination problem and thus no functional pathologies. This is, of course, to assume away the problem of who adjusts prices in such a way that information is complete. In the case of strategic interaction complete information is conveyed by the idea of “common knowledge”, but interaction will then have important consequences and the outcomes in an economy with this sort of interaction will not, in general, be Pareto optimal. However, putting to one side the idea of strategic behavior, interaction, once introduced, involves non-linearity and externalities and, more generally, one loses the proportionality between cause and effect: small shocks may lead to large effects.

In our opinion, we must take into account the different nature of atoms and agents: this involves going beyond the strict confines of the methodology proposed in statistical physics and the passage to the economics of complexity: “Imagine how hard physics would be if atoms could think”, Murray Gell-Mann, reported by Page (1999). The main consequence of learning, or of learning atoms as stated in the quotation of Gell-Mann, is that the tools of physics cannot be translated sic et simpliciter in economics. This does not mean that the tools should be abandoned, but does mean that one has to specify carefully the way in which previous experience is translated into current choices. The contributions of Foley (1994) and Aoki (1996) introduced into economics the possibility of treating the issue of many heterogeneous and interacting agents analytically and thus to micro-found aggregate behavior without recurring to the heroic but scientifically unacceptable hypothesis of a representative agent. With this achievement we depart from the economics of mechanical equilibrium of the Poinsot-Walras setting to out of economics of equilibrium probability distributions, in which the single agent can find herself outside of equilibrium and the system becomes complex.

A crucial aspect of the complexity approach is how interacting elements produce aggregate patterns that those elements in turn react to. This leads to the emergence of aggregate properties and structures that cannot be guessed by simply looking at individual behavior. It has been argued (Saari, 1995) that complexity is ubiquitous in economic problems (although this is rarely acknowledged in economic modeling), since (i) the economy is inherently characterized by the direct interaction of individuals, and economics in particular. After having derived analytic functions, they model learning agents as a finite memory Markov chain, and subsequently derive the corresponding master equation that describes the evolution of the population of the same agents. On modeling ABM as Markov chains (see Gintis, 2012).

Note also that the possibility of interaction between agents can address the problem of coordination failures (that is, for example, crises and fluctuations).

(ii) these individuals have cognitive abilities, e.g. they form expectations about aggregate outcomes and base their behavior upon these expectations.\(^7\)

In a nutshell, the passage from economics to economic complexity will be characterized by (we will argue in the following):

- the passage from an axiomatic discipline (which is actually what economics is) toward a falsifiable science (falsifiable at different levels of aggregation);
- economic policies in which one size does not fit all.

2. From classical mechanics economics to...

2.1.

The current crisis has moved a large number, if not the majority of economists, to reflect on the state of economic theory. Unreliable economic models have not provoked the crisis, although some authorities such as Lord Turner have argued that the economics profession does indeed bear a large part of the responsibility for the current crisis. As he says:

“But there is also a strong belief, which I share, that bad or rather over-simplistic and overconfident economics helped create the crisis. There was a dominant conventional wisdom that markets were always rational and self-equilibrating…”

Adair Turner, Head of the UK Financial Services Authority (2010)

This complacency led economists to continue to build models, which have not been able to provide measures to prevent and certainly not to forecast a crisis such as the present one. In particular, those reductionist models populated by a perfectly rational and fully informed representative agent turned out to be extremely fallacious (Kirman, 2009; Colander et al., 2009; but see Lucas, 2003 and Blanchard, 2009). Although the standard approach is still predominant, despite its apparent internal coherence, its ability to explain the empirical evidence is increasingly questioned.

The causes of the present state of economic theory date back to the mid of the XVIII century (Mirowski, 1989), when some of the Western economies were transformed by the technological progress which lead to the industrial revolution. This was one century after the Newtonian revolution in physics: from the small apple to the enormous planets, all objects seemed to obey the simple natural law of gravitation. It was

\(^7\) Agent Based complexity theory should not be confused with general systems theory, a holistic approach developed in the 1950s and 1960s that in its most radical form argued that everything affects everything else: according to systems theory “phenomena that appear to have simple causes, such as unemployment, actually have a variety of complex causes – complex in the sense that the causes are interrelated, nonlinear, and difficult to determine” (Phelan, 2001). Conversely, the complexity approach looks for simple rules that underpin complexity.
therefore an obvious choice for a new type of social scientist, the economist, to borrow the method (mathematics) of the most successful hard science, physics, allowing for the mutation of political economy into economics. It was (and still is) the mechanical physics of the XVI century, which rules and has ruled economics. Since then, economics has lived its own evolution based on the assumptions of classical physics (reductionist, deterministic and mechanistic).

The concepts of natural laws and equilibrium have been transplanted into economics sic et simpliciter. As a consequence of the adoption of the classical mechanics paradigm, the difference between micro and macro was analyzed from a reductionist standpoint. In such a setting, aggregation is simply the process of summing up market outcomes of individual entities to obtain economy-wide totals. This means that there is no difference between micro and macro: the dynamics of the whole is nothing but a summation of the dynamics of its components. This approach does not take into consideration that there might be two-way interdependencies between the agents and the aggregate properties of the system: interacting elements produce aggregate patterns to which those elements, in turn, react, what are called the positive and negative feedbacks in the economy. Soros has drawn attention to the importance of these effects and has coined the term “reflexivity”. What macroeconomists typically fail to realize is that the correct procedure of aggregation is not a sum: this is where emergence (i.e. the arising of complex structures from simple individual rules: von Hayek, 1948; Schelling, 1978) enters the scene. Physics taught us that considering the whole, as something more than its constitutive parts is not only a theoretical construction: it is a description of reality. Empirical evidence, as well as experimental tests, shows that aggregation generates regularities, i.e. simple individual rules, when aggregated, produce statistical regularities or well-shaped aggregate functions: regularities emerge from individual “chaos” (Lavoie, 1989).

The research program launched by the neoclassical school states that macroeconomics should be explicitly grounded on micro-foundations. According to the standard approach, this implies that economic phenomena at a macroscopic level should be explained as being based on the choices made by independent individual decision makers.

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8 The concept of equilibrium is quite a dramatic example. In many economic models equilibrium is described as a state in which aggregate demand equals supply. The notion of statistical equilibrium, in which the aggregate equilibrium is compatible with individual disequilibrium, is outside the box of tools of the standard economist. The same is true for the notion of evolutionary equilibrium (at an aggregate level) developed in biology, according to which an individual organism is in equilibrium only when it is dead. The equilibrium of a system no longer requires that every single element be in equilibrium by itself, but rather that the statistical distributions describing aggregate phenomena be stable, i.e. in “[…] a state of macroscopic equilibrium maintained by a large number of transitions in opposite directions” (Feller, 1957: 356).

9 “The most interesting recent developments in macroeconomic theory seem to us describable as the incorporation of aggregative problems […] within the general framework of ‘microeconomic’ theory. If these developments succeed, the term ‘macroeconomic’ will simply disappear from use and the modifier ‘micro’ will become superfluous. We will simply speak, as did Smith [sic!], Marshall and Walras, of economic theory” (Lucas, 1987: 107–108).
Furthermore, the independence of the agents is used to justify treating the aggregate choices as the sum of individual choices, which are subject only to technical and budget constraints. The reduction of the degree of freedom, which is characteristic of the aggregation problem in physics, is ruled out: a rational agent with complete information can choose to implement his individually optimal behavior, without any additional constraints imposed by his interaction with others.

There are three main pillars of this approach:

- The precepts of the rational choice-theoretic tradition;
- The equilibrium concept of Walrasian analysis; and
- The reductionist approach of classical physics.

The first two, which constitute the justification for reducing macro to micro, are logically flawed (and empirically unfounded). As is well established, the standard, and very restrictive, assumptions on individuals do not allow one to assume that the aggregate will behave like an individual. Rejection of the third, however, leaves open the road to a different approach that of complexity.

Standard economic theory, like theology, is an axiomatic discipline. According to the supporters of this view, such an abstraction is necessary since the real world is complicated: rather than compromising the epistemic worth of economics, simplifying assumptions are essential to advance economic knowledge, and this, of course, is true for any modeling approach. However, this argument does not invalidate the criticism of unrealistic assumptions (Rappaport, 1996). While it requires internal coherence, so that theorems can be logically deduced from a set of assumptions, it abstracts from external coherence between theoretical statements and empirical evidence. This suggests that economics has made, an important epistemological detachment from falsifiable sciences like physics. In a sense, the position taken by economic theorists resembles that taken by pure mathematicians as the following observation by Bourbaki, the group on whose work much of Debreu’s contributions to economics was based:

“Why do applications [of mathematics] ever succeed? Why is a certain amount of logical reasoning occasionally helpful in practical life? Why have some of the most intricate theories in mathematics become an indispensable tool to the modern physicist, to the engineer, and to the manufacturer of atom-bombs? Fortunately for us, the mathematician does not feel called upon to answer such questions.”

Bourbaki Journal of Symbolic Logic 1949, 2

2.2.

In setting the methodological stage for the Dynamic Stochastic General Equilibrium (DSGE) macroeconomic theory, Lucas and Sargent declared: “An economy following a multivariate stochastic process is now routinely described as being in equilibrium, by which is meant nothing more that at each point in time (a) markets clears and (b)
agents act in their own self-interest. This development, which stemmed mainly from the work of Arrow [...], and Debreu [...], implies that simply to look at any economic time series and conclude that it is disequilibrium phenomenon is a meaningless observation. [...] The key elements of these models are that agents are rational, reacting to policy changes in a way which is in their best interests privately, and that the impulses which trigger business fluctuations are mainly unanticipated shocks.” (Lucas & Sargent, 1979: 7).

The self-regulating order (now it would be called SOC: Self Organization Criticality) of Adam Smith (1776) is transformed into a competitive General Equilibrium (GE) in the form elaborated in the 1870s by Walras, that is a configuration of prices and plans of action such that, at those prices, all agents can carry out their chosen plans and, when they do so, markets clear. In a continuous effort to achieve greater generalization and analytical sophistication, modern (neoclassical) economists interested in building microfoundations for macroeconomics soon turned to the most refined version of the general equilibrium model, that proposed in the 1950s by Arrow & Debreu (1954). There they introduced uncertainty into the model and showed that individual intertemporal optimization yields a GE, as soon as the individuals in the economy are equipped with perfect price foresight for each future state of nature and a complete set of Arrow-securities markets (Arrow, 1964), all open at time zero and closed simultaneously. Of course, the model was open to the objection that since the number of commodities was assumed to be finite this meant that, at some point the economy terminated. This was resolved by Bewley (1972) who introduced an infinite dimensional commodity space. However, here again realism is lost since this means that one assumes that individuals live for ever. Their task is, to say the least, unreasonable, since they have only one budget constraint which then extends forever also. However, ignoring these difficulties, whenever the conditions we have just outlined hold true, the GE is an allocation that maximizes a properly defined social welfare function, or, in other words, the equilibrium is Pareto-efficient (First Welfare Theorem).

The literature has pointed out several logical inconsistencies of the standard approach to economic theory. Davis (2006) identifies three “impossibility results” which he suggests epitomize the breakdown of standard, i.e. neoclassical, economics:

- Arrow’s 1951 theorem showing that there is no mapping from individual preferences to social preferences consistent with some simple axioms, a clear indication of the aggregation problem (Arrow, 1963);
- The Cambridge capital debate pointing out that standard theory is contradictory with respect to the concept of aggregate capital and the assumptions on technology (Cohen & Harcourt, 2003); and
- The Sonnenschein (1972), Mantel (1974), Debreu (1974) results showing that the standard comparative static reasoning is inapplicable in general equilibrium models, i.e. that GE is neither unique nor locally stable under general conditions.
Moreover, by construction, in a GE all transactions are undertaken at the same equilibrium price vector. Regardless of the mechanism (Walras’ or Edgeworth’s assumption) one adopts, the GE model assumes that the formation of prices precedes the process of exchange, instead of being the result of it, through a tatonnement process occurring in a meta-time. Real markets work the other way round and operate in real time, so that the GE model cannot be considered as a realistic explanation of real economic phenomena (Arrow, 1959). Furthermore, no explanation is given as to why there should be a unique price for each good or how such a unique price is attained.

It has been widely recognized since Debreu (1959) that integrating money in the theory of value represented by the GE model, is at best, problematic. Given that in a GE model actual transactions take place only after a price vector coordinating all trading plans has been freely found, money can be consistently introduced into the picture only if the logical keystone of the absence of transaction costs is abandoned. By the same token, since credit makes sense only if agents can sign contracts in which one side promises delivery of goods or services to the other side at a time other than that at which payment is made in equilibrium, in the Arrow-Debreu sense, markets for debt are meaningless. Individuals are not faced with the problem of synchronizing purchase and payment since their only budget constraint is a lifetime one. Abstraction is made from the whole structure necessary for such transactions and information conditions and information processing requirements are not properly defined, and bankruptcy can be safely ignored.

Put alternatively, the very absence of money and credit is a consequence of the fact that in GE there is complete information and rationality, i.e. there is no time. The only role assigned to time in a GE model is, in fact, that of dating commodities. Products and their characteristics, technologies, and preferences are exogenously given and fixed from the outset. The convenient implication of banning out-of-equilibrium transactions is simply that of getting rid of any disturbing influence of intermediary modifications of endowments – and therefore of individual excess demands – on the final equilibrium outcome.

Although the RA framework has a long history, it became standard to use this device to build macroeconomic models on micro-foundations only after Lucas’ famous critique paper (1976). Standard models are characterized by an explicitly stated optimization problem of the RA, which is used to obtain the “aggregate” demand or supply curves. These are assumed to be obtained from the derived individual demand or supply curves. Even when the models allow for heterogeneity, interaction is generally absent or assumed to have no impact (the so-called weak interaction hypothesis: Rioss Rull, 1995). The use of RA models should avoid the Lucas critique, provide micro-foundations for macroeconomics, and, çà va sans dire, justify continuing to build Walrasian general equilibrium models.
Since models with many heterogeneous interacting agents are complicated, macroeconomists assume the existence of a RA: a simplification that makes it easier to solve for the competitive equilibrium allocation, since direct interaction is ruled out by definitions. Unfortunately, as Hildenbrand & Kirman (1988) noted: “There are no assumptions on isolated individuals, which will give us the properties of aggregate behavior. We are reduced to making assumptions at the aggregate level, which cannot be justified, by the usual individualistic assumptions. This problem is usually avoided in the macroeconomic literature by assuming that the economy behaves like an individual. Such an assumption cannot be justified in the context of the standard model.”

The equilibria of general equilibrium models with a RA are characterized by a complete absence of trade and exchange, which is a counterfactual idea. Kirman (1992), Gallegati (1993) and Caballero (1992) show that RA models ignore valid aggregation concerns, by ignoring interaction and emergence, committing fallacy of composition (what in philosophy is called the “fallacy of division”, i.e. to attribute properties to a different level from which the property is observed: game theory offers a good case in point with the concept of Nash equilibrium, by assuming that social regularities come from the agent level equilibrium). Those authors provide examples in which the RA does not represent the individuals in the economy so that the reduction of a group of heterogeneous agents to RA is not just an analytical convenience, but “both unjustified and leads to conclusions which are usually misleading and often wrong” (Kirman, 1992).

A further result, which is a proof of the logical fallacy in bridging the micro to the macro is the impossibility theorem of Arrow: it shows that an ensemble of people, which has to collectively take a decision, cannot show the same rationality of an individual unless one individual is a dictator in which case society respects his preferences alone (Mas-Colell, Whinston, & Green, 1995). Moreover, the standard econometric tools are based upon the assumption of a RA. If the economic system is populated by heterogeneous (even without their necessarily interacting) agents, then the problem of the micro-foundation of macro econometrics becomes a central topic, since some issues (e.g., co-integration, Granger-causality, impulse-response function of structural VAR) lose their significance (Forni & Lippi, 1997). Moreover as Forni and Lippi point out one can reproduce complicated aggregate behavior as the collective result of individuals with rather simple behavior. In other words the complexity of aggregate behavior reflects the diversity of simple individuals not the behavior of individuals who are themselves complex.

All in all, we might say that the failure of the RA framework points out the vacuum of the standard micro-foundation literature, which ignores interactions: no toolbox is available to connect the micro and the macro levels, beside the RA whose existence is at odds with the empirical evidence (Stoker, 1995; Blundell & Stoker, 2005) and the equilibrium theory as well (Kirman, 1992).
3. …the ABM approach

3.1.

What characterizes a complex system is the notion of emergence, that is the spontaneous formation of self-organized structures at different layers of a hierarchically configured system (Crutchfield, 1994). Standard economics, on the other hand conceptualizes the economic system as consisting of several identical and isolated components, each one being a copy of a RA. The aggregate solution can thus be obtained by means of a simple summation of the choices made by each optimizing agent. In this way it becomes clear that the RA device, is simply a way of avoiding the problem of aggregation by eliminating heterogeneity. But by doing this one cannot eliminate the underlying heterogeneity in the real system. If the macroeconomist takes this seriously, he has to derive aggregate quantities and their relationships from the analysis of the micro-behavior of different agents. This is exactly the key point of the complexity approach: starting from the micro-equations describing/representing the (optimal) choices of the economic units, what can we say about the macro-equations? Do they have the same functional form as the micro-equations (the analogy principle)? If not, how should one derive macro-theory?

However, it is hard to recognize the inheritance of methodological individualism in the RA paradigm, which claims that the whole society can be analyzed in terms of the behavior of a single representative individual and forgets to apply to that individual the Lucas critique. On the other hand, focusing on aggregate phenomena arising “from the bottom up” (Epstein & Axtell, 1996) via the interaction of many different agents, ABM adopts a holistic approach when it claims that these phenomena cannot be studied without looking at the entire context in which they are embedded. Indeed, holism is the idea that all the properties of a given system cannot be determined or explained by the sum of its component parts alone as Simon (1969) did not tire of pointing out. Instead, the system as a whole determines in an important way that the parts behave. He indicated clearly the relationship between reductionism and holism when he said:

“Roughly, by a complex system I mean one made up of a large number of parts that interact in a non simple way. In such systems, the whole is more than the sum of the parts, not in an ultimate metaphysical sense, but in the important pragmatic sense that, given the properties of the parts and the laws of their interaction, it is not a trivial matter to infer the properties of the whole. In the face of complexity, an in-principle reductionist may be at the same time a pragmatic holist.”

Herbert Simon, 1969: 267

From this perspective, ABM can be regarded as a bridge between methodological individualism and methodological holism. In ABM models aggregate outcomes (the

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10 The general principle of holism was concisely summarized by Aristotle in his *Metaphysics*: “The whole is more than the sum of its parts”, a manifesto of the complexity approach.
“whole”, e.g. the unemployment rate) are computed as the sum of individual characteristics (its “parts”, e.g. individual employment status). However, aggregate behavior can often be recognized as distinct from the behavior of the agents, which make up the whole, leading to the discovery of emergent properties (Dosi, Fagiolo, & Roventini, 2010; Gallegati et al., 2010). In this sense, the whole is more than – and different from – the sum of its parts. It might even be the case that the whole appears to act as if it followed a distinct logic, with its own goals and means, as in the example of a cartel of firms that act in order to influence the goods market price. From the outside, the “whole” appears no different from a new agent type (e.g. a family, a firm). The computational experiment that gives birth to a new entity has been successful in “growing artificial societies from the bottom up” (Epstein & Axtell, 1996).

This bottom-up approach to complexity consists in “deducing the macroscopic objects (macros) and their phenomenological complex ad-hoc laws in terms of a multitude of elementary microscopic objects (micros) interacting by simple fundamental laws” (Solomon, 2007), and ABM provides a technique that allows to systematically follow the birth of this complex macroscopic phenomenology. The macros at a specific scale can become the micros at the next scale.

The ABM methodology is from bottom-up and is focused on the interaction of many heterogeneous agents, which might produce a statistical equilibrium (Miller & Page, 2006; Epstein, 2006b; see also Batten, 2000; Wooldridge, 2001; Gilbert & Troitzsch, 2005 and Flake, 1998).

ABM is a methodology which allows one to construct models with heterogeneous agents, based on simple behavioral rules and on the interaction between these agents, where the resulting aggregate dynamics and empirical regularities are not known a priori and are not deducible from individual behavior (Nicolis, G., & Nicolis, C., 2007). ABM is characterized by two main tenets: (i) there is a multitude of objects that interact with each other and with the environment; (ii) the objects are autonomous, i.e. there is no central, or “top down” control over their behavior.

The bottom-up approach models individual behavior according to simple behavioral rules; agents are allowed to have local interaction and to change their individual rules (through adaptation) as well as the network which governs their interaction. By aggregating, some statistical regularity emerges, which cannot be inferred from individual behavior (self emerging regularities); this emergent behavior feeds back to the individual level (downward causation) thus establishing a macro foundation of micro behavior. As a consequence, each and every proposition may be falsified at micro, meso and macro levels. This approach is in direct opposition to the axiomatic theory of economics, where the optimization of isolated and independent agents is the standard for a scientific, i.e. not ad-hoc, modeling procedure.11

11 In the spirit of Simon the agent-based methodology can also be viewed as a way to reconcile the two opposing philosophical perspectives of methodological individualism and holism. Having agents as unit of analysis, ABM is deeply rooted in methodological individualism, a philosophical method aimed at
3.2.

Agent-based computational economics (ACE) is the area of computational economics that studies economic processes, including whole economies, as dynamic systems of interacting agents. As such, it falls in the paradigm of complex adaptive systems. In corresponding agent-based models, the “agents” are “computational objects modeled as interacting according to rules” over space and time, not real people. The rules are formulated to model behavior and social interactions based on incentives and information. This is in stark contrast to the principle enshrined by Lucas when he argued that one should make no other assumptions in economics other than those on individual characteristics. He said specifically:

“Now, I am attracted to the view that it is useful, in a general way, to be hostile toward theorists bearing free parameters, so that I am sympathetic to the idea of simply capitalizing this opinion and calling it a Principle”

Lucas, 1980: 709

In insisting on this Lucas locked his macroeconomic colleagues into a very restrictive framework. However, later he was to observe:

“Applications of economic theory to market or group behavior require assumptions about the mode of interaction among agents as well as about individual behavior.”

Lucas, 1988

In the sort of models corresponding to the complexity approach something more is going on. The underlying assumptions on individual behavior are and can be weakened because of the interaction between agents. The theoretical assumption of mathematical optimization by agents in equilibrium is replaced by the less restrictive postulate of agents with bounded rationality adapting to market forces. ACE models apply numerical methods of analysis to computer-based simulations of complex dynamic problems for which more conventional approaches, such as the proof of theorems under very restrictive assumptions may not find ready use. Starting from initial conditions specified by the modeler, the computational economy evolves over time as its constituent agents repeatedly interact with each other, including learning from interactions. In these respects, ACE has been characterized as a bottom-up culture-dish approach to the study of economic systems.

explaining and understanding broad society-wide developments as the aggregation of decisions by individuals (von Mises, 1949; Arrow, 1994). Methodological individualism suggests – in its most extreme (and erroneous) version – that a system can be understood by analyzing separately its constituents, the reductionist approach that the “whole” is nothing but the “sum of its parts” (Descartes, 1637; Nagel, 1961). However, the ability to reduce everything to simple fundamental objects and laws does not imply the ability to start from those objects and laws and reconstruct the universe. In other terms, reductionism does not imply constructionism (Anderson, 1972).
The outcome of interaction is numerically computed. Since the interacting objects are autonomous, they are called “agents”: “Agent-based Computational Economics is the computational study of economic processes modeled as dynamic systems of interacting agents” (Tesfatsion, 2002, 2006; Gintis, 2007; Chen, 2012).

Here “agent” refers broadly to a bundle of data and behavioral methods representing a constitutive part of a computationally constructed world. The availability of high-speed processors and the possibility to handle large amounts of data has undoubtedly contributed to the success of ACE models. One of the problems detected is related to parameter setting: with many degrees of freedom, as is often objected, any result becomes possible. The process of aggregation, in physics, takes away these degrees of freedom. The micro-founded approach to macroeconomics is very different from that used in physics. The latter starts from the micro-dynamics of the single particle, as expressed by the Liouville equation and, through the Master equation, ends up with the macroscopic equations. In the aggregation process, the dynamics of the agents lose their degrees of freedom and behave coherently in the aggregate. In standard economics, while the procedure is formally the same (from micro to macro), it is assumed that the dynamics of the agents are those of the aggregate. The reduction of the degrees of freedom, which is characteristic of the aggregation problem in physics, is therefore ruled out: a rational agent with complete information can choose to implement the individually optimal behavior, without additional constraints.

### 3.3.

Following the example of statistical physics, Foley (1994), Aoki (1996, 2002), Aoki & Yoshikawa (2006) have proposed to aggregate statistically HIA. In this way, one can obtain an analytical solution for populations of millions of agents that interact. Figure 1 represents the share of non self financing firms in the same total firm population – 1 million firms vs. 1 Master Equation, (ME) in the model of Landini et al. (2012a) – described respectively by the methods of ACE and of ASHIA. The correlation between the two is 0.87: note that the approximation results in an acceptable discrepancy, especially given the reduction in the number of equations: from 1 million to 1!

Until now, the analyses of systems with HIA have been limited to simulations. The ASHIA approach follows a quite different methodology, adopting analytical tools originally developed in statistical mechanics, and subsequently adopted by social disciplines. As the economy is populated by a very large number of heterogeneous and interacting agents, we cannot know which agent is in which condition at a given time and whether an agent will change its condition, but we can know the probability of a given state of the world. The basic idea consists of introducing a level of aggregation, obtained by grouping the agents in clusters according to a measurable variable. This state variable is

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12 See also Di Guilmi, Gallegati, & Landini (2011); Landini et al. (2012a, 2012b); Gallegati et al. (2006); Lux (2009).
chosen in such a way that the dynamics of the number of individuals in each cluster also define the evolution of the whole economy.

This study is made possible by specifying some general assumptions on the stochastic evolution of the occupation numbers, which can be modeled by means of the master equation (ME). The latter is a simple first-order differential equation, which quantifies the evolution through time of the probability of observing a given number of agents in a certain state. The process of reducing the vector of observations of a variable over a population to a single value (i.e. the computation of an average level of output for each dimensional bin) is defined as mean field approximation. The definitions of the mean-field variables and of the probabilities involve some level of interaction among agents.\textsuperscript{13}

The ME allows us to derive an analytic device, which could have a strong impact on macroeconomic modeling. It proposes a solution to the problem of performing the aggregation when heterogeneity and nonlinearities are present, an issue, which is debated in the literature at least since the introduction of exact aggregation. In fact, the ME is an analytical tool that allows us to interpret the aggregate dynamics by means of a (mechanical) statistical approach, that integrates the heterogeneity by considering a multiplicity of representative states of a system, and implements the interaction (as a mean field interaction) by means of the specification of the transition rates. In such a way, the problem of the aggregation of heterogeneous agents is originally solved, without re-

\textsuperscript{13} Mean-field theory has been introduced in economics in different models by a number of authors including Brock and Durlauf, who show how mean-field interaction is able to generate a multiplicity of Nash-type equilibria and Weisbuch, Kirman, & Herreiner (2000) who study the evolution of the relations in the buyer seller network in the Marseille fish market.
sorting either to the unrealistic simplifications and assumptions of standard economic theory, or to the “black box” of computer simulations as in the standard ACE approach. Furthermore this methodology can effectively deal with the issue of agents’ interaction, by functionally embodying it in the determination of the probabilistic transition rules (the transition rates), which change endogenously in time. As one can see, this development changes macroeconomic modeling: the empirical evidence can directly induce the representation of the micro-level, and the task of the researcher is just to identify a suitable set of relationships among the micro-variables.\(^\text{14}\)

Moreover, as demonstrated by the physics literature, the ME can be effectively employed to model analytically the dynamics of the topology of a network. Landini et al. (2012a), for instance, analyses the evolution of the degree distribution of a network. The solution of the ME yields the dynamics of the network degree, providing a synthetic and formal representation of the concentration in the market and, thus, of the fragility of the network.

Summarizing, the main contributions of ASHIA are:

- To provide a solution to the problem of aggregation in HIAs by using methods inspired by the statistical mechanics as opposed to the usual classical mechanics involved in RA based models;
- To propose a dynamic stochastic model for sub-populations of many HIAs interacting in an endogenous evolving network;
- To develop an analytic solution in which the model is defined as a deterministic ordinary differential equation describing the dynamics of network.

The dynamic stochastic aggregation is able to provide a complete and consistent analytical representation of the system, using numerical simulations only as a further verification of the results of the model.

Economic agents are not atoms, however: they have many more choices according to their preferences and endowments, but mostly according to rationality and information. There is only one-way to match \textit{homo oeconomicus} with an atom: perfect information and complete rationality. A person not with a sound mind can behave in a totally unpredictable way, while even a poorly informed can guess. In this process of “trials and errors” lies the process of learning, often modeled by complexity theorists, such as CAS.

Apart from the equivalence between atom and \textit{homo oeconomicus}, we have to consider some of the consequences of introducing these methods into economics, among which, in our opinion, the following three points are the most relevant:

- With learning, the contribution of ASHIA profoundly changes: it is no longer the mere transposition into economics of an instrument borrowed from statistical

\(^\text{14}\) It is worth noticing that this approach overcomes another limit of the RA modeling, as the equilibrium is no more a fixed point in the space, but a probability distribution: a system can be in equilibrium even if its constitutive elements are not.
physics, since it is transformed in the direction of scientifically studying the society in which individual behavior changes and it is changed by the aggregate context; the notion of reflexivity is important here;

- To appreciate the systemic aspect of an economy, one has to analyze the individual as well the global characteristics of the system by not only analyzing the individual nodes as agents but also the way they are interlinked in their network;

- The social aspect of the economic atom is expressed in networks: the links between agents are established (e.g. to form credit linkages and/or to increase the information set) and are changed according to fitness (see Albert & Barabási, 2000; Bianconi & Barabási, 2001; Tedeschi, Iori, & Gallegati, 2012).

As we have said within the standard macroeconomic paradigm, there is no direct interaction among economic units (for a pioneer, though neglected, contribution see Foellmer, 1974; see also Kirman, 2000). In the most extreme case, any individual strategy is excluded (principle of excluded strategy, according to Schumpeter, 1960) and agents are homogeneous. Small departures from the perfect information hypothesis are incoherent with the Arrow-Debreu general equilibrium model, as shown by Grossman & Stiglitz (1976), since they open the chance of having direct links among agents (Stiglitz, 1992). In particular, if prices convey information about the quality, there cannot be an equilibrium price as determined by the demand-supply schedule, since demand curves depend on the probability distribution of the supply (Grossman & Stiglitz, 1976: 98).

4. The role of economic policy

Economics was previously called political economy. According to classical economists, the economic “science” should be used to control the real economies and steer them towards desirable outcomes. If one considers the economic system as an analogue of the physical one, it is quite obvious to look for natural economic policy prescriptions (one policy fits all). This is the approach of standard (neoclassical) economists.

There is a widespread opinion, well summarized by Brock & Colander (2000), that complexity does not add anything new to the toolbox of standard economic policy analysis. This view needs substantial correction (see also the reflections by Durlauf, 1997). The complexity approach showed us that the age of certainty ended with the non-equilibrium revolution, exemplified by the works of Prigogine. Considering the economy as an evolving (adaptive) system we have to admit that our understanding is limited: there is no room for the Laplace’ demon in complexity. Individual behavioral rules evolve according to their past performance: this provides a mechanism for an endogenous change of the environment. As a consequence the “rational expectation hypothesis” loses its significance. However, agents are still rational in that they do what they can in order not to commit systematic errors (Lewontin & Levins, 2008). In this setting there is still room for policy intervention outside the standard myth of a neutral and opti-
Figure 2. Distribution of firms’ trade-credit relations in the electronic-equipment sector in Japan, 2003 (De Masi et al., 2010).

Real economies are composed of millions of interacting agents, whose distribution of characteristics is far from being purely stochastic or normally distributed. As an example, Figure 2 reports the distribution of firms’ trade-credit relations in the electronic-equipment sector in Japan in 2003 (see De Masi et al., 2010). It is quite evident that there exists several hubs, i.e. firms with many connections: the distribution of the degree of connectivity is scale free, i.e. there are a lot of firms with 1 or 2 links, and quite a few firms with a lot of connections. Let us assume the Central Authority has to prevent a financial collapse of the system, or the spreading of a financial crisis (the so-called domino effect, see e.g. Krugman, 1998 and Stiglitz, 2002). Rather than looking at the “average” risk of bankruptcy (in power law distributions the mean may even not exist, i.e. there is an empirical mean, but it is not stable), using the latter as a measure of the stability of the system by means of a network analysis, the economy can be analyzed in terms of different interacting sub-systems and local intervention can be recommended to prevent failures and their spread.\textsuperscript{15} Instead of a helicopter drop of liquidity, one can make “targeted” interventions to a given agent or sector of activity.

In this perspective, notions elaborated from network theory become very relevant, like resilience, which depicts the behavior of network’s structures following the removal of some nodes. In particular, whenever a vertex is removed from a network, the av-

\textsuperscript{15} For instance, Fujiwara (2008) shows how to calculate the probability of going bankrupt either alone, i.e. because of idiosyncratic elements, or because of a domino effect, i.e. because of the failure or other agents with which there exist credit or commercial links.
average distance among nodes increases and, as this process goes further, some nodes will ultimately be disconnected. Nodes can be removed in many ways. They may be attacked randomly or according to some of their intrinsic properties (such as their degree). Depending on the rules used to remove nodes, the network shows a different level of resilience. For instance, Albert & Barabási (2000) show that social networks, usually highly right-skewed, are remarkably resistant to random attacks but extremely vulnerable to attacks targeted at nodes with the highest degree (hubs). To prove this claim, the authors remove nodes in decreasing order of their connectivity, showing that, as a small number of hubs are removed, the average distance of the scale-free network increases rapidly.

Network topology is relevant for systemic risk too. Credit relationships, which have acted as a major channel of contagion during the crisis, can be naturally conceived as networks in which nodes represent agents and links represent credit claims and liabilities. In particular, it becomes important to identify densely connected subsets of nodes within such networks, i.e. modules or communities. In fact, community structure is tightly related to the issue of diversification because, in a nutshell, the latter may be attained only where the former is suppressed. Since instead communities are likely to be ubiquitous in real economic networks, community detection provides a general approach to the analysis of contagious defaults. In fact, contagion is dependent on the geometric properties of the network with adjacency matrix representing (possibly weighted) connections, which on its part is related to the community structure of the network (Bargigli & Gallegati, 2012). Thus a community detection algorithm provides a general recipe to detect those areas of the financial system, which are most likely to be affected when some nodes are initially hit by shocks, without the need to specify in advance such shocks (for a general method of detecting communities see Copic, Jackson, & Kirman, 2009).

As another example and one of considerable importance at the present time, and in order to understand why the structure of the network is important it is worth looking at the international financial network. Figures 3 to 5 show the evolution of the international financial network from 1985 to 2005.

In this case, the nodes correspond to countries and the size of the nodes to the total amount of foreign assets held by the country corresponding to the node in question. A link between countries means that at least one of the two holds the assets of the other. Typically one would define a minimum threshold for such assets to constitute the basis for a link. The thickness or weight of the link represents the sum of the mutually held assets. Once these definitions are established, one can calculate the empirical degree distribution and one can see what proportion of the total weight of all the links is made up by the total of the weights associated with the links emanating from the largest nodes. What we know is that, while the connectivity of the global financial network has increased remarkably in recent years (see Nier et al., 2007), the degree distribution 3 has changed and has become more skewed with a few nodes having very high degree and a group of nodes becoming very central. To quote Haldane (2009), of the Bank
Figure 3. The Global Financial Network in 1985 (source Haldane, 2009).

Figure 4. The Global Financial Network in 1995 (source Haldane, 2009).
of England, when talking about these developments in the banking network before the global financial crisis, he says:

“This evolution in the topology of the network meant that sharp discontinuities in the financial system were an accident waiting to happen. The present crisis is the materialization of that accident.”

Haldane, 2009: 4

In a heterogeneous interacting agents environment, there is also room for an extension of the Lucas critique. It is well known that, since the underlying parameters are not policy invariant, any policy advice derived from large-scale econometric models that lack micro-foundations would be misleading. The Lucas Critique implies that in order to predict the effect of a policy experiment, the so-called deep parameters (preferences, technology and resource constraints) that govern individual behavior have to be modeled. Only in this case it is possible to predict the behavior of individuals, conditional on the change in policy, and aggregate them to calculate the macroeconomic outcome. But here is the trick: aggregation is a sum only if interaction is ignored. If non-price interactions (or other non-linearities) are important, then the interaction between agents may produce very different outcomes. Standard economic models focus on analytical solvable solutions: to get them, they have to simplify the assumptions, e.g. using the RA approach or a Gaussian representation of heterogeneity. In the end, the main objective
of these models is to fit the theory, not the empirical: how to explain, e.g., the scale-
free network of the real economy by using the non-interacting network of the standard
model? At a minimum, one should recognize that the standard approach is a very prim-
itive framework and, as a consequence, the economic policy recommendations derived
from it are far from being adequate prescriptions for the real world.

One of the traditional fields of applications of economic policy is redistribution. It
should be clear that a sound policy analysis requires a framework built without the
RA straight jacket. A redistributive economic policy has to take into account that indi-
viduals are different: not only because they behave differently, e.g. with respect to sav-
ing propensities, but also they have different wealth: the so-called St. Matthew (13:12)
effect (“to anyone who has, more will be given and he will grow rich; from anyone
who has not, even what he has will be taken away”), which is the road to Paradise for
Catholics, and to the power-law distribution of income and wealth for the econophysi-
cists.

Gaffeo et al. (2007) shows that there is a robust link between firms’ size distribu-
tion, their growth rate, and GDP growth (see also Gabaix, 2011). This link determines
the distributions of the amplitude frequency, size of recessions and expansion, etc. Ag-
gregate firms’ size distribution can be well approximated by a power law (Axtell, 2001;
Gaffeo, Gallegati, & Palestini, 2003), while sector distribution is still right skewed,
but without scale-free characteristics (Axtell, 2001). Firms’ growth rates are far from
being normal: in the central part of the distribution they are tent shaped with very fat
tails. Moreover, empirical evidence shows that an inverse function of firms’ age and size
exists, and is proportional to financial fragility. In order to reduce the volatility of fluc-
tuations, policy makers should act on the firms’ size distribution, allowing for a growth
of their capitalization, their financial solidity and wealth redistribution (Delli Gatti et al.,
2004, 2005). Since these emerging facts are policy sensitive, if the aggregate parameters
change the shape of the curve will shift as well. Note also, as Gabaix (2011) indicates,
once we take account of the power law distribution of firm sizes in the US for exam-
ple, an idiosyncratic shock to a big firm can have major macroeconomic repercussions.
Thus the typical macroeconomic story of unexplained and unidentified aggregate shocks
becomes unnecessary.

As opposed to Keynesian economic policy, which develops aggregate economic
policy tools without micro-foundations, and standard neoclassical economics, which
prescribes individual incentives because of the Lucas critique but ignores interaction
which is a major but still neglected part of that critique, the ABM approach proposes a
bottom-up analysis. What generally comes out is not a “one-size-fits-all” policy since it
depends on the general as well as the idiosyncratic economic conditions; moreover, it
generally has to be conducted at different levels (from micro to meso to macro). In short,
ABM can offer new answers to old unresolved questions, although it is still in a far too
premature stage to be considered as a complete alternative paradigm.
5. Future directions

Depending on the purpose and scope of the analysis, it is generally useful to stop at some scale in way down from macro to micro to reconstruct aggregate and top-level dynamics “from the bottom up”. In the case of economics, rather few levels (e.g. a micro, a meso and a macro level) are in general sufficient to provide a thorough understanding of the system. Defining the elementary units of analysis amounts to fixing the limits for the reductionist approach, which is not a priori discarded but rather integrated into the analysis. These units are in fact characterized by an inner structure that does not depend on the environment in which they are embedded. They can thus be analyzed separately.

The need for the ABM approach at any given scale is often linked to the existence of some underlying autocatalytic process at a lower level. Autocatalytic processes are dynamic processes with positive feedbacks, where the growth of some quantity is to some extent self-perpetuating, as in the case when it is proportional to its initial value.\footnote{The importance of positive feedback has been recognized in the literature on increasing returns, in particular with respect to the possibility of multiple equilibria (Semmler, 2005), since the time of Marshall. It should also be remembered that despite various valiant efforts, in general, one couldn’t even prove the existence of equilibrium in the presence of non-convexities.} However, the traditional analysis is static, and does not address how one equilibrium, if there is one, out of several might be selected. Looking at the problem from the perspective of dynamic stochastic process, selection is explained in terms of one set of small historical events magnified by increasing returns.

Moreover, the existence of an autocatalytic process implies that looking at the average, or most probable, behavior of the constituent units is not representative of the dynamics of the system: “autocatalyticity insures that the behavior of the entire system is dominated by the elements with the highest auto-catalytic growth rate rather than by the typical or average element” (Solomon, 2007). In presence of autocatalytic processes, even a small amount of individual heterogeneity invalidates any description of the behavior of the system in terms of its “average” element: “the real world is controlled as much by the tails of distributions as by means or averages. We need to free ourselves from average thinking” (Anderson, 1972).

The fact that autocatalytic dynamics are scale invariant (i.e. after a transformation that multiplies all the variables by a common factor) is a key to understanding the emergence of scale invariant distributions of these variables (e.g. power laws), at an aggregate level. The relevance of scale free distributions in economics (e.g. of firm size, wealth, income, etc.) is now extensively recognized (Brock, 1999), and has been the subject of through investigation in the econophysics literature (Mantegna & Stanley, 2000).

In the 4th edition of his Principles, Marshall wrote, “The Mecca of the economist is biology”. What he clearly had in mind was that, because economics deals with learning agents, evolution and change are the granum salis of our economic world. A
ory built upon the issue of allocations of given quantities is not well equipped for the analysis of change. This allocation can be optimal only if there are no externalities (increasing returns, non-price interactions etc.) and information is complete, as the invisible hand shows. In the history of science, there is a passage from a view emphasizing centralized “intelligent design” to a view emphasizing self organized criticality (Bak, 1997), according to which a system with many heterogeneous agents reaches a statistical aggregate equilibrium, characterized by the appearance of some (often scale free) stable distributions. These distributions are no longer “optimal” or “efficient” according to some welfare criterion: they are simply the natural outcome of individual interaction. Schumpeter with his theory of “creative destruction” clearly took this position.

As a result of the internal and external inconsistencies of the standard approach to economics already mentioned, a growing strand of economists is now following a different methodology based upon the analysis of systems with many heterogeneous interacting agents. Their interaction leads to empirical regularities, which emerge from the system as a whole and cannot be identified by looking at any single agent in isolation: these emerging properties are, according to us, the main distinguishing feature of a complex system. The focus on interaction allows the scientist to abandon the heroic and unrealistic RA framework, in favor of the science of complexity. The complexity approach is a very challenging line of research whose empirical results are very promising (see e.g., chapters 2–3 in Gaffeo et al., 2007). Modeling an agent-based economy however remains a complex and complicated adventure.

References


